Towards Memory-Efficient Incremental Processing of Streaming Graphs

by

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B.Sc., Shahid Beheshti University, 2018

Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

in the
School of Computing Science
Faculty of Applied Sciences

SIMON FRASER UNIVERSITY
Spring 2021

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Abstract

With growing interest in efficiently analyzing dynamic graphs, streaming graph processing systems rely on stateful iterative models where they track the intermediate state as execution progresses in order to incrementally adjust the results upon graph mutation to reflect the changes in the latest version of the graph. We observe that the intermediate state tracked by these stateful iterative models significantly increases the memory footprint of these systems, which limits their scalability on large graphs. Due to the ever-increasing size of real-world graphs, it is crucial to develop solutions that actively limit their memory footprint while still delivering the benefits of incremental processing.

We develop memory-efficient stateful iterative models that demand much less memory capacity to efficiently process streaming graphs with delivering the same results as provided by existing stateful iterative models. First, we propose a *Selective Stateful Iterative Model* where the memory footprint is controlled by selecting a small portion of the intermediate state to be maintained throughout execution, and the selection can be configured based on the capacity of the system's memory. Then, we propose a *Minimal Stateful Iterative Model* that further reduces the memory footprint by exploiting the key properties of graph algorithms. We develop incremental processing strategies for both of our models in order to correctly compute the effects of graph mutations on the final results even when intermediate states are not available. The evaluation shows our memory-efficient models are effective in limiting the memory footprint while still retaining most of the performance benefits of traditional stateful iterative models, hence being able to scale on larger graphs that could not be handled by the traditional models.

Keywords: Streaming Graph; Graph Processing; Incremental Computation; Dynamic Graph

Dedication

For my family

Acknowledgements

Throughout my MSc studies, I have received a great deal of support and encouragement from many people.

I would like to thank my advisor, Prof. Keval Vora, for his support and invaluable expertise in guiding me throughout this project. Your insightful feedback pushed me to bring my work to a higher level.

I am very grateful to all of the members of my thesis committee. Prof. Nick Sumner, Prof. Tianzheng Wang and Dr. Ouldooz Baghban Karimi, thank you for your collaboration.

I like to thank all of my lab members: Lynus, Rakesh, Kasra, Matthew, Joanna, Mio, and especially Mugilan for the assistance and cooperation.

I am very thankful to all of my friends: Arman, Ariyan, Amirhossein, Amirali, Pooya, Emad, Hossein, Moeen, Amin, Saba, Mehryar, MohamadReza, Reza, Ali, Parmida and a especial thank goes to Hamid for supporting and motivating me during my master studies.

Last but not least, I would like to express my gratitude to my family. This thesis would not have been possible without your unconditional love and support. I am who I am today only because of you.

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Chapter 1

Introduction

Dynamic graphs are common across various application domains like social network analysis [7], bioinformatics [12], machine learning [16] and web analysis [5]. Streaming graph processing systems [39, 38, 48, 26, 11, 21] aim to deliver real-time results as the graph structure changes via a continuous stream of edge and vertex mutations.These systems are equipped with efficient iterative processing models that are broadly classified into two types: the *stateless iterative model* and the *stateful iterative model*.

Stateless iterative models [39, 11, 38, 21] perform iterative processing without capturing additional intermediate values that describe the execution history. When the graph structure mutates (e.g., new edges/vertices get added or old vertices/edges get removed), they either continue the iterative computation without correctly incorporating the impact of mutations on already computed values (i.e., not guaranteeing accuracy of final results) [39]; or, they conservatively deduce the set of values that could be potentially affected due to mutation (using techniques like tag propagation [38]) and recompute those values from scratch.

Stateful iterative models used in [48, 26, 25] capture the intermediate state (representing execution history) as computation progresses, often in terms of intermediate values computed for vertices in each iteration. When the graph structure mutates, only the change in values resulting from those mutations are iteratively propagated to correct the intermediate state and compute accurate final results. As expected, stateful iterative models are more efficient in generating correct final results compared to stateless iterative models simply because the stateful models operate on the precise set of intermediate values that get affected by the change. Stateless iterative models, on the other hand, need to be conservative while generating accurate results since they do not capture intermediate values representing the execution history, and hence, they end up demanding much more computation.

The amount of intermediate state saved by the stateful iterative models depends on the nature of the graph algorithms they support. For example, models that operate on asynchronous graph analytics algorithms like BFS, shortest paths, connected components, etc. leverage the monotonic relationship of the values in the algorithm to adjust them directly [48], which requires capturing only $O(|V|)$ intermediate state. On the other hand,

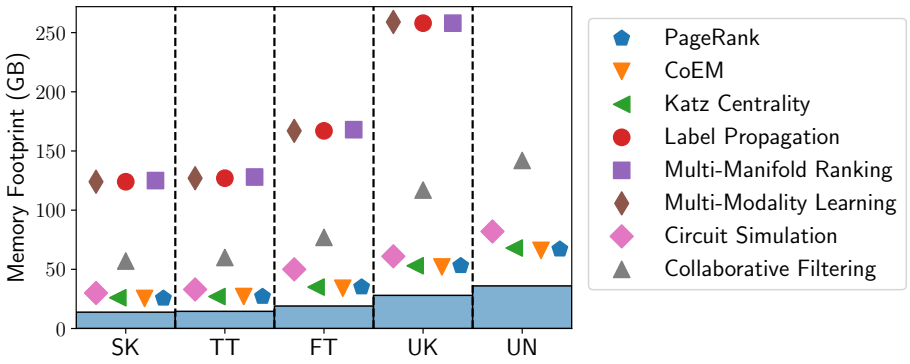


Figure 1.1: Memory footprint of stateful iterative model across different graph algorithms (shown as different points) and graph datasets (details in Table 6.1). Solid bars represent the memory consumed by the graph structure.

models that support synchronous graph algorithms like Co-Training Expectation Maximization (CoEM), Collaborative Filtering (CF), etc. capture the intermediate state at every iteration in order to incrementally recompute the values iteration-by-iteration and guarantee results equivalent to Bulk Synchronous Parallel (BSP) [44] execution from scratch [26]. Furthermore, the intermediate state in each iteration often contains the aggregation results for vertices along with the vertex values, and their size (in bytes) depends on the graph algorithm being run. For example, CoEM requires 8 bytes per intermediate state of a vertex whereas CF consumes 16 bytes per intermediate state. Other graph algorithms like Multi-Modality Learning (MML) and Label Propagation (LP) have even larger intermediate states depending on the number of labels they operate on.

We profiled GraphBolt [26], a recent state-of-the-art streaming graph processing system, to measure the amount of memory consumed by the intermediate state across different graph algorithms and graph datasets. As shown in Figure 1.1, the intermediate state consumes at least twice the amount of memory required to hold the input graph itself; for the Collaborative Filtering algorithm this factor increases to over 4× whereas the intermediate state in algorithms like Multi-Manifold Ranking and Label Propagation consumes over an order of magnitude more memory compared to the input graph.

Such high memory footprint significantly limits the scalability of the stateful iterative models on large graphs. For instance we observed that three of the graph algorithms in Figure 1.1 ran out of memory for the UN graph mainly because those executions ran out of the available memory capacity (320GB). With graph datasets growing at a faster rate than memory capacity, it becomes crucial to develop stateful iterative models that do not demand large memory capacities just to hold the intermediate states.

1.1 Contributions

In this dissertation, we develop memory-efficient stateful iterative models that demand much less memory capacity to efficiently process streaming graphs and provide the same BSP guarantees as existing state-of-the-art streaming graph processing systems. First, we present a *Selective Stateful Iterative Model* where the memory footprint is controlled by selecting a small portion of the intermediate state to be maintained during execution. And then, we present a *Minimal Stateful Iterative Model* that specializes the incremental processing for certain graph algorithms (depending on their update functions) to drastically reduce the memory footprint.

1.1.1 Selective Stateful Iterative Model

The key insight here is that vertex computations within an iteration are independent of each other, and hence, incrementally recomputing the value of a vertex is only dependent on the intermediate state saved for that vertex (i.e., not dependent on the states saved for other vertices). Moreover, the usefulness of different portions of the intermediate state (in terms of the amount of computation pruned out upon graph mutation) is different; this means, intermediate states for certain vertices would end up reducing more computation than those for other vertices.

Based on these insights, our selective stateful iterative model tunes its memory footprint depending on the available main memory by selecting the intermediate vertex states to be captured at a fine-grained level (illustrated in Figure 1.2). While this enables the model to scale on large streaming graphs, performing incremental computation using the partial intermediate state becomes challenging. This is because the value changes resulting from graph mutations are typically merged with the intermediate vertex states to propagate subsequent (transitive) changes throughout the graph, and how to compute the effects of value changes with missing intermediate states remains unclear. To address this, we develop a *selective incremental processing technique* that correctly computes the effects of changes even when intermediate vertex states are not available. Our strategy captures the nuances of the interaction between vertices with intermediate states and those without intermediate states to ensure that the latter set can correctly participate in the iteration-by-iteration incremental computation.

1.1.2 Minimal Stateful Iterative Model

In this model, we specialize the incremental processing for certain algorithms. Specifically, algorithms like CoEM and PageRank involve operations that are purely distributive, i.e., outgoing value changes from a given vertex can be directly computed based on the incoming value changes to the vertex. For such algorithms, we identify that the effects of graph mutations can be propagated iteration-by-iteration throughout the graph even without using

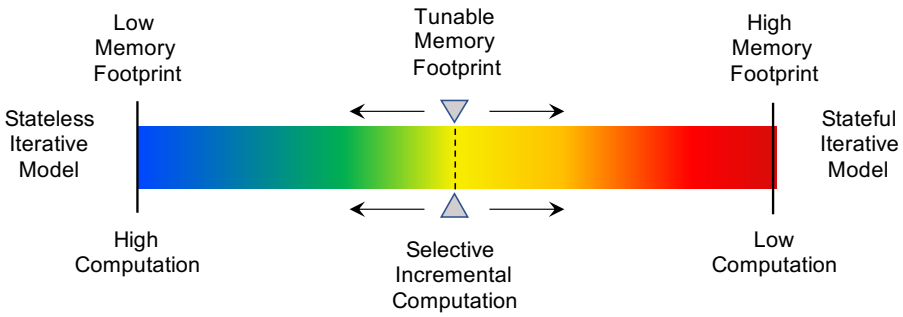


Figure 1.2: Sliding scale of memory and computation requirements between stateless iterative model and stateful iterative model. By capturing only partial intermediate state (selectively at a fine-grained level), memory consumption can be reduced at the cost of increased computation.

the intermediate states for most of the vertices. Specifically, only the intermediate states for vertices that get directly affected by graph mutation are needed to perform incremental processing over the entire graph.

We use this insight to develop the minimal stateful iterative model, where the amount of intermediate state gets aggressively reduced to a known subset of vertices on which mutations take place, hence consuming a much smaller memory footprint that is dependent on the graph mutations instead of the original graph size. The incremental computation strategy upon graph mutation retains the direct computation of changes for vertices without intermediate states along with carefully adjusting the available intermediate states iteration-by-iteration for vertices directly impacted by mutation.

1.1.3 Overview of Results

Our proposed techniques are general, and can be incorporated in any streaming graph processing system to leverage incremental processing without incurring high memory footprint. We implemented both of our proposed memory-efficient models in GraphBolt [26]; since GraphBolt's existing stateful iterative model maintains intermediate states for all the vertices in the graph, it becomes a natural baseline to demonstrate the effectiveness of our proposed models.

Our evaluation with five real-world graphs and seven synchronous graph algorithms shows that our models demand significantly less memory capacity in comparison to GraphBolt, while still retaining most of its performance benefits. Specifically, the memory footprint of our selective stateful iterative model is dependent on the amount of intermediate state saved; for instance, by selecting only 20% of the intermediate state to be saved, the memory footprint is 35-70% smaller than that for GraphBolt. Furthermore, our minimal stateful iterative model reduces the memory footprint by 28-58% even when it is used for algorithms that have smaller intermediate state to begin with. This allows our memory-efficient stateful

iterative models to scale to very large graphs that could not be handled by GraphBolt with the available memory capacity. Results from this dissertation are published in [45].

1.2 Dissertation Outline

The dissertation is organized as follows. Chapter 2 presents the background on stateless/stateful iterative models and chapter 3 highlights the related works in dynamic graph processing. Chapter 4 explains the *Selecive Stateful Iterative model* as well as vertex tracking and adjusting the states. Chapter 5 describes the *Minimal Stateful Iterative model*. The experimental evaluation with memory consumption and execution time results of our proposed techniques are described in chapter 6. Finally, the conclusion and future directions are outlined in chapter 7.

Chapter 2

Background

In this chapter, we review the streaming graph processing model and the stateful iterative processing that performs incremental computation.

2.1 Streaming Graph Processing

A *streaming graph* is a graph whose structure keeps on changing via a continuous stream of graph updates (e.g., addition and deletion of vertices and edges). The change in graph structure is also referred to as mutation of graph structure, and each individual update arriving from the stream is also called a mutation. There are many challenges in this area including efficient ingestion of the mutation along with answering the user queries, storing the evolving dataset and the programming model. *Streaming graph processing systems* [48, 26, 6, 25] operate on streaming graphs to continuously produce results consistent with the latest graph structure. To do so, they often rely on *incremental processing* techniques where the computed results for the previous graph snapshot are adjusted based on how the graph structure mutates.

In this dissertation, we propose memory-efficient stateful iterative models to incrementally process the streaming graphs using bulk synchronous parallel semantics.

Synchronous Processing Semantics

The Bulk Synchronous Parallel (BSP) model [44] is a popular model used to design parallel algorithms where parallel computations are separated out across different iterations (or super-steps). Each iteration is divided into two phases: the computation phase where threads operate on data in parallel, and the communication phase where threads exchange updated results with each other. This enforces a clear separation between between the values being computed/written and those being read, which enables programmers to easily analyze the convergence and correctness guarantees of parallel algorithms. In this thesis, we focus on algorithms that leverage BSP processing.

Algorithm 1 BSP Processing Model for PageRank

1: $G = (V, E)$
2: $pr = [\frac{1}{|V|}, \frac{1}{|V|}, ...]$
3: **while** not converged **do**
4: $newPr = [0, 0, ...]$
5: **parallel for** $(u, v) \in E$ **do**
6: $AtomicAdd(\&newPr[v], \frac{pr[u]}{outDegree(u)})$
7: **end parallel for**
8: **parallel for** $v \in V$ **do**
9: $newPr[v] = 0.15 + 0.85 \times newPr[v]$
10: **end parallel for**
11: $swap(pr, newPr)$
12: **end while**

Many graph algorithms can be easily expressed using the BSP model. Algorithm 1 shows the PageRank algorithm implemented with the BSP model. The vertices are processed iteratively such that in each iteration, the read set and the write set are kept disjoint. For each vertex, the neighboring values are read from *pr* variable (lines 5-7), and the newly pagerank value is written to *newPr* (lines 8-10). At the end of each iteration, *newPr* and *pr* are swapped (line 11) to make the newly computed values visible in the next iteration.

Incremental Computation

While continuous stream of graph updates get rapidly applied to the graph, the overall structure of the graph often changes gradually over time. Hence, recomputing the results from scratch every time upon graph update becomes unnecessary as the previously computed results are often useful in quickly computing the final results for the version after graph gets updated. This is achieved using incremental computation.

Incremental computation reuses the results that were calculated before the mutation of the graph so that only the inconsistent portion of the results is recomputed incrementally, not the entire result set. Incremental computation often requires tracking some intermediate state that reflects the execution history, so that necessary history can be replayed correctly. Depending on whether streaming graph processing solutions track intermediate states or not, we categorize them into two kinds: *stateless iterative processing models* and *stateful iterative processing models.*

2.1.1 Stateless Iterative Processing Model

Stateless iterative processing models [39, 11, 38, 21] perform regular iterative processing without capturing additional intermediate values that describe the execution history. When the graph structure mutates (e.g., new edges/vertices get added or old vertices/edges get removed), they either continue the iterative computation without correctly incorporating

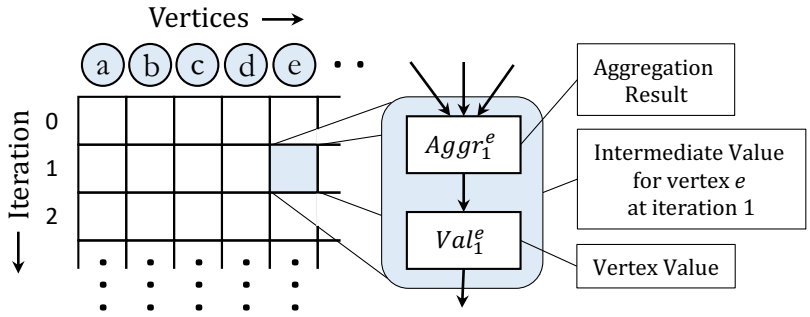


Figure 2.1: Intermediate state in terms of values relevant for vertices in each iteration. Each intermediate value consists of the aggregation value (to incrementally merge differences) and the vertex value (to compute outgoing differences).

the impact of mutations on already computed values (i.e., not guaranteeing accuracy of final results) [39]; or, they conservatively deduce the set of values that could be potentially affected due to mutation (using techniques like tag propagation [38]) and recompute those values from scratch.

2.1.2 Stateful Iterative Processing Model

Stateful iterative processing models used in recent systems like [48, 26, 6, 25] reduce the amount of computation to be performed upon graph mutation using incremental processing. The main idea is to track the intermediate state that captures the necessary details of execution history so that when the graph structure mutates, only the relevant parts of execution history are adjusted or recomputed.

To guarantee end results that are same as a BSP execution starting from scratch, GraphBolt employs a *dependency-driven incremental refinement* strategy [26] which tracks the vertex values in each iteration and then upon graph mutation, it incrementally recomputes only the affected values by propagating changes or *differences* in values occurring as a result of graph structure mutation. Specifically, when the graph structure mutates, affected vertices and edges (i.e., ones that got mutated) are activated to propagate missing old values (for edge additions) and retract old values (for edge deletions) which are used to adjust the tracked values for target vertices. Then, changes in values of those target vertices are iteratively propagated to the rest of the graph to incrementally adjust the intermediate state and vertex values. Changes are propagated in iteration-by-iteration manner, so that correctness guarantees (in form of BSP semantics) are retained for every iteration all the way till the end, hence ensuring that the final result are same as computed by a BSP execution from scratch.

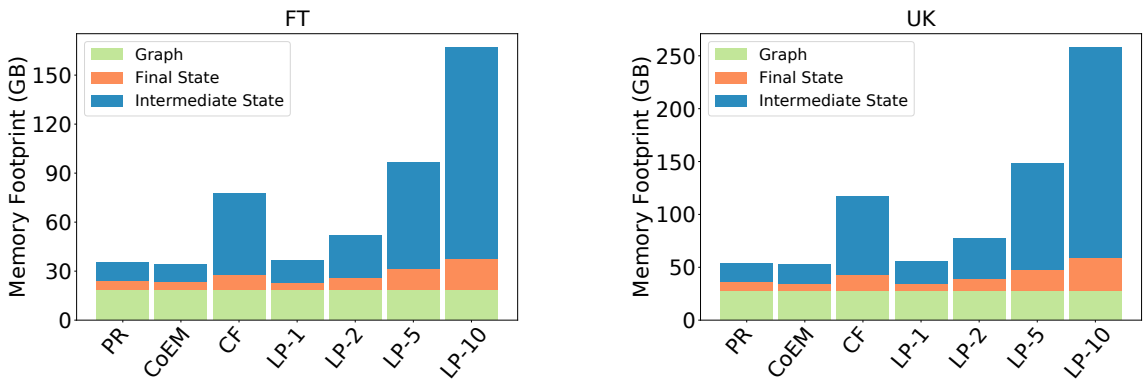


Figure 2.2: Memory consumption of different components in stateful iterative model: graph structure, final vertex results, and intermediate state. The intermediate state requires different amount of memory across different graph algorithms. On Label Propagation, the memory consumed by intermediate state increases as number of labels increase (indicated by LP-k where k is the number of labels).

2.2 Tracking Intermediate State in Memory

The intermediate state in stateful iterative models is in form of values relevant for vertices at each iteration. As shown in Figure 2.1, these intermediate values often consist of two components: first, the result of aggregation (also called *aggregation value*) at each vertex that collects values from its incoming edges; and second, the value computed for that vertex. For our PagerRank example in Algorithm 1, the intermediate state would contain the result of *newPr* at line 6 and *pr* value for each vertex. Maintaining both of these values as intermediate state becomes crucial because iterative algorithms often use selective scheduling mechanisms in order to suppress propagation of minor changes (e.g., change less than `1e-2` threshold). In such cases, the outgoing vertex value for a given iteration may not be based on its aggregation result since the latter can hold multiple minor changes which may not have been propagated to the outgoing neighbors. By explicitly tracking the two values, differences can be correctly computed and propagated based on values visible to the neighbors.

As expected, tracking this intermediate state increases the memory footprint of such incremental processing techniques. While Figure 1.1 shows the high memory footprint for different graph algorithms on graph datasets, Figure 2.2 compares the size of intermediate state relative to the remaining memory consumption of the process (majority of which is taken by the input graph structure and then vertex frontiers). Even though PageRank and CoEM operate on scalar values, their intermediate states, and final states consume nearly as much memory as the remainder of the processes. Collaborative Filtering operates on two factors per vertex (i.e., its vertex state is a size-2 vector), and hence, its intermediate state ends up consuming up to 80% additional memory compared to the stateless execution. Finally, Label Propagation operates on feature vectors; as shown in Figure 2.2, increasing the

number of features directly increases the amount of memory consumed by the intermediate state. In fact, maintaining intermediate states with 10 features requires 129GB additional memory for FT graph and 198GB additional memory for UK graph (graph details in Table 6.1), which increases the memory footprint by 3.44× and 3.3× respectively compared to their stateless executions. Such high amount of memory consumption significantly limits the applicability of the stateful iterative processing model on large graphs.

Chapter 3

Related Work

Several dynamic graph processing techniques have been developed in the literature. We first discuss the solutions with stateful iterative models and then summarize the works that use stateless models. Ultimately, we briefly discuss generalized streaming platforms.

3.1 Stateful Iterative Processing

Kickstarter [48], GraphBolt [26] and DZiG [25] develop efficient streaming graph processing solutions using stateful iterative models for incremental computation. Upon graph mutation, these systems use the intermediate state to quickly adjust the computed values and deliver final results corresponding to the latest graph version.

KickStarter [48] focuses on graph algorithms like BFS and SSSP that use monotonic functions. Its runtime exploits the monotonic relationship between vertex values to capture dependencies between the latest computed values, resulting in only one intermediate state per vertex. Hence, its memory footprint does not drastically increase compared to stateless execution of those algorithms as the input graph consumes the most amount of memory.

GraphBolt [26] and DZiG [25] (built in GraphBolt) focus on the broader class of graph algorithms that run in BSP manner. They capture the dependency information across intermediate vertex values as computation progresses, and not just across the latest values. Hence, the intermediate state requires much larger amount of memory which drastically increases their memory footprint as shown in Figure 2.2. Our memory-efficient stateful iterative models limit the memory footprint by reducing the amount of intermediate state that gets captured for efficient incremental computation.

GraphInc [6] is another system that uses stateful iterative model. It captures all the messages between vertices as part of the intermediate state along with intermediate values at vertices. This means the amount of intermediate state tracked by GraphInc is in the order of edges, which is significantly higher than that tracked by GraphBolt and DZiG (which is in order of vertices). Hence, GraphInc's memory footprint is typically orders of magnitude higher than GraphBolt and DZiG, making it an impractical solution for large graphs.

3.2 Stateless Iterative Processing

Systems like [39, 9, 21, 11, 13, 38, 42, 24] use stateless iterative models which limits their efficiency in delivering accurate results upon graph mutation. Tornado [39], Kineograph [9] and GraphIn [38] perform incremental computation by triggering the user functions based on graph updates and allowing the changes to propagate throughout the graph. Hence, they cannot guarantee accurate results for BSP algorithms. GraphIn identifies the vertices that could be potentially impacted by graph updates using tag propagation, and restarts computation from scratch for those identified vertices. [42] uses GIM-V (generalized iterative matrix vector multiplication) to perform incremental computation. LLAMA [24], STINGER [13], Aspen [11], GraphOne [21] and LiveGraph [58] focus on designing efficient dynamic graph data structures and storage systems, and their processing models do not support incremental computation.

Other solutions [17, 18, 28, 47] operate on evolving graphs that contain a group of temporally-related graph snapshots capturing the evolution of the graph structure over time. This is different from streaming graphs where a single graph snapshot is continuously mutated. These systems do not capture intermediate states and perform incremental computation by directly reusing the results computed for previous graph snapshots, making them suitable for self-fixing graph algorithms but not BSP algorithms.

Finally, static graph processing systems [40, 31, 57, 14, 35, 15, 34, 53, 22, 37, 29, 46, 49, 51, 50] can be used to compute results for the latest graph version by simply throwing away the results upon graph mutation and restarting the computation from scratch. As expected, such a solution delivers low performance [26].

3.3 Generalized Streaming Solutions

Generalized streaming systems [1, 2, 36] provided a general programming model for streaming different format of data that can be used for streaming graph processing. Naiad [30] presents a timely dataflow model that enables stateful iterative and incremental computation, and Differential Dataflow [27] introduce *differential computation* by extending incremental computation in timely data flow. [54] is a real-time query processing solution that presents a splitting operator to stream high volume inputs by breaking them into sub streams. Apache Flink [8] supports streaming and batch computation by tracking intermediate states.

Chapter 4

Selective Stateful Iterative Model

In this chapter, we develop a *selective incremental processing* model that tracks intermediate states only for a selected subset of vertices that demand more computations.

4.1 Intuition & Main Idea

Maintaining intermediate state essentially allows incremental processing where the effects of graph mutation are propagated in form of value changes throughout the graph. On the other extreme when intermediate state is not maintained, vertex values that are recomputed in a given iteration have to be pushed out since there is no way to determine whether the new values are different from ones computed prior to graph mutation. We observe that every single vertex computation, either in incremental manner with intermediate state or from scratch without intermediate state, is a local computation. This means the value for a given vertex can be computed as long as the right values arrive from its in-neighbors (either in form of value differences or actual values). Hence, we selectively trade off the benefits of incremental computation with reduced memory footprint at a fine-grained level.

Our selective stateful iterative model tracks the intermediate states of only a subset of vertices instead of all the vertices in the graph. For vertices whose intermediate states are not tracked, the model reconstructs their states on-the-fly so that changes resulting from graph mutation can be directly propagated. As illustrated in Figure 1.2, this allows us to limit the memory footprint by directly controlling the subset of vertices whose intermediate values are tracked, at the cost of performing more computation for vertices whose intermediate states are not tracked.

For simplicity, the vertices whose intermediate states are tracked are called *tracked vertices* whereas the remaining vertices are called *untracked vertices*.

4.2 Tracking Useful Vertex States

In order to selectively track intermediate states, we need to answer two main questions: first, how many vertices should be tracked, and second, which specific vertices should be tracked.

4.2.1 How many vertices should be tracked?

This can be directly answered based on the memory capacity (or budget) assigned for the process. Specifically, the size of the intermediate vertex states (which is algorithm dependent) can be automatically determined during initialization, which can be used to bound the number of vertices to be tracked using the available memory budget:

$$mem\_budget \geq k \times state\_size \times t + base\_size$$

where k is the number of vertices whose states are tracked, mem_budget is the available memory capacity, $state\_size$ is the size of intermediate vertex state, t is the number of iterations for which intermediate state should be captured, and $base\_size$ is the memory consumed by other data structures in the system (e.g., input graph structure, vertex frontiers, stream buffers, etc.) along with additional capacity for the graph structure to grow over time. Hence, the above equation can be rewritten to maximize the number of tracked vertices k as:

$$\underset{k}{\text{argmin}} \ |mem\_budget - (k \times state\_size \times t + base\_size)|$$

Note that the majority of $base\_size$ is consumed by the graph data structure, whereas the remaining structures like vertex frontiers (which are simply boolean arrays) often consume less than 10% memory compared to the graph data structure.

4.2.2 Which vertices should be tracked?

A naive way to select vertices to be tracked can be using random sampling, where tracking of intermediate states can be enabled for a random subset of vertices. While such a strategy easily allows selecting vertices, it remains oblivious of how incremental processing gets performed, and hence it fails to maximize the benefits of incremental processing. Since different vertices require different amount of computation depending on how values get propagated throughout the graph, we must ideally select those vertices that demand high computation so that most of their computation can be effectively eliminated by incremental processing. To do so, we consider the 'usefulness' of the intermediate state for each vertex, where the usefulness of an intermediate state is informally defined as the amount of computation it ends up reducing for that vertex. The usefulness of an intermediate state depends on several dynamic factors including the distance (in terms of number of hops) from the vertices

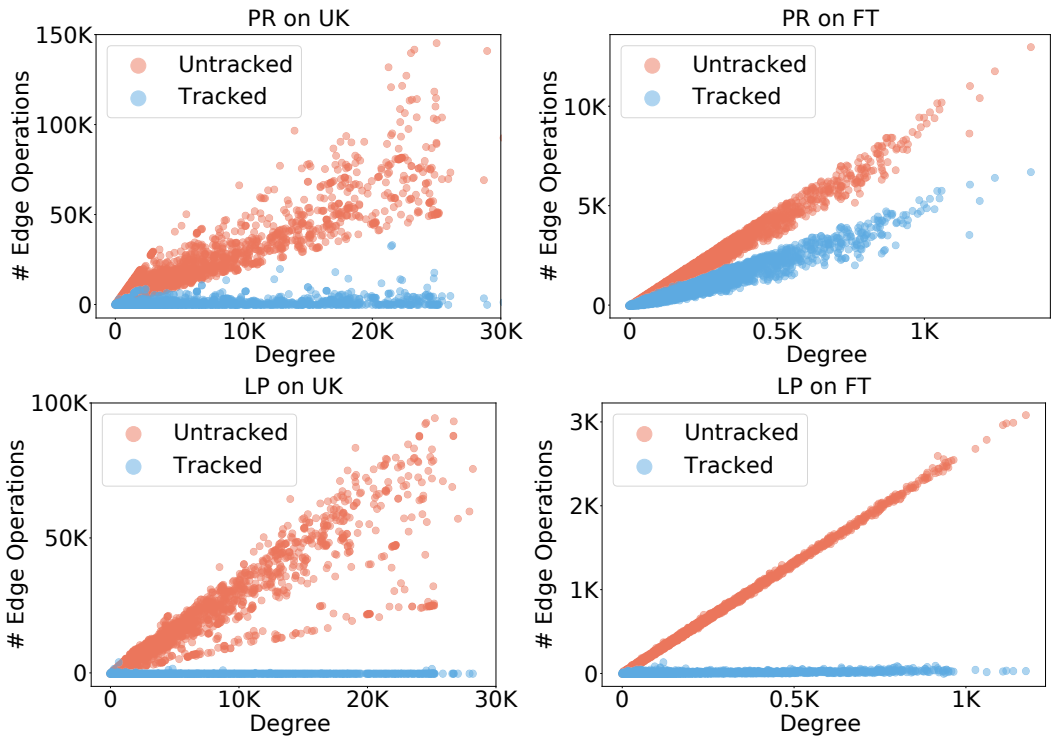


Figure 4.1: Number of edge operations performed for tracked and untracked vertices based on their in-degrees. Tracking high in-degree vertices reduces more edge operations compared to tracking low in-degree vertices.

where mutations got applied, and the sensitivity of the graph algorithm to changes in graph structure. Since accurately computing such a metric is infeasible for the general case, we approximate the usefulness of an intermediate state using a vertex-local heuristic.

To develop our heuristic, we profiled the amount of computation performed on each vertex when processing a given graph snapshot. The computation for each vertex is measured in terms of the number of edge operations performed for that vertex; this is because edge operations are expensive (often involve atomic writes and random accesses) and they are the primary candidates that incremental processing attempts to reduce [26]. Figure 4.1 correlates the number of edge operations for different vertices with their in-degrees for two executions: first, the execution where all vertices' intermediate states are tracked (i.e., GraphBolt's dependency-driven incremental processing); and second, the execution where computation is started from scratch (i.e., no intermediate state is tracked). As we can see vertices with higher in-degree demand more computation when their intermediate values are not tracked, and tracking their values reduces their computation requirements. For instance, the top 20% of the high in-degree vertices contribute to up to 34.1-94.9% of the total number of edge operations in Figure 4.1. Hence, tracking the intermediate states for high in-degree vertices is most useful. On the other hand, the savings for low in-degree vertices are small (visible from the gap between orange points and blue points for low in-degree vertices)

since those vertices demand fewer computation even when their intermediate states are not tracked.

Therefore, we track the intermediate states for top-k vertices ranked with highest in-degree. The top-k vertices get selected using a linear pass over vertices when the graph snapshot gets initialized; the vertex ids whose intermediate states must be tracked are maintained in a k-sized buffer. Furthermore, as execution progresses, the subset of tracked vertices can be incrementally adjusted to eliminate certain vertices and add new vertices whenever graph mutations significantly impact the top-k vertex ranking. This is achieved incrementally by recomputing the intermediate values for vertices that get newly added in the top-k list, so that effects of subsequent graph mutations get handled incrementally for the new vertices.

Discussion. Most real-world graphs have skewed degree distributions, and certain graphs like road networks have a more uniform degree distribution where the spread between the highest and lowest degrees is small. While tracking higher degree vertices would work for uniform degree distribution as well, stochastic selection can also be utilized to gain reasonable performance without managing degree-based selection. Furthermore, our selection process does not assume any distribution of graph mutations or their impact on the intermediate values. If incoming mutations are expected to affect known regions of the graph, then the selection process can be enhanced to track vertices from those regions.

4.3 Incremental Processing upon Mutation

With intermediate states available for only a subset of vertices, propagating changes resulting from graph mutations becomes challenging. This is because values during the incremental refinement stage can flow across different vertices regardless of whether their intermediate states are tracked or not. Since we aim to guarantee BSP semantics, computation of vertex values cannot be deferred as they need to happen in an iteration-by-iteration manner.

We develop a *selective incremental processing* technique that operates on selective intermediate states, i.e., where a selected subset of vertices are tracked. Our technique effectively separates out the interactions between tracked and untracked vertices so that right values get propagated across the edges depending on whether their source and target vertices are tracked or untracked. We first summarize how the graph layout can be optimized when selective intermediate states are maintained, and then discuss the details of our selective incremental processing.

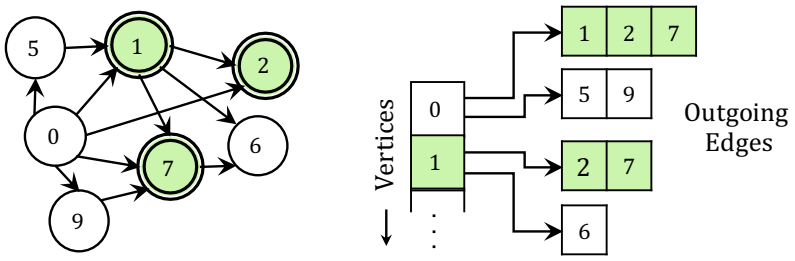


Figure 4.2: Optimized graph layout. Vertices 1, 2 and 7 are tracked (highlighted), and remaining vertices are not tracked. Each vertex maintains two adjacency lists for outgoing edges: one for tracked neighbors and other for untracked neighbors.

4.3.1 Optimizing Graph Layout

Streaming graph processing systems use efficient dynamic graph representations to handle rapid graph mutation and enable efficient parallel operations on active edges and vertices [13, 24, 11, 26]. Since our selective incremental processing handles the interactions for tracked vertices in a different manner compared to the interactions for untracked vertices, the graph layouts can be improved to avoid expensive checks while propagating values during the refinement process. Specifically, the edges between tracked vertices and untracked vertices can be separated out in the graph layout itself; by doing so, all computations on edges whose target vertices are either tracked or untracked can be performed directly in form of parallel operations without verifying whether every individual target vertex is tracked or untracked.

Since we incorporate our technique in GraphBolt, which uses adjacency lists to hold graph snapshots, such a separation results in the graph layout shown in Figure 4.2. Here, each vertex now holds two vectors for its outgoing neighbors: the first vector contains edges whose targets are tracked vertices, and the second vector contains edges whose targets are untracked vertices.

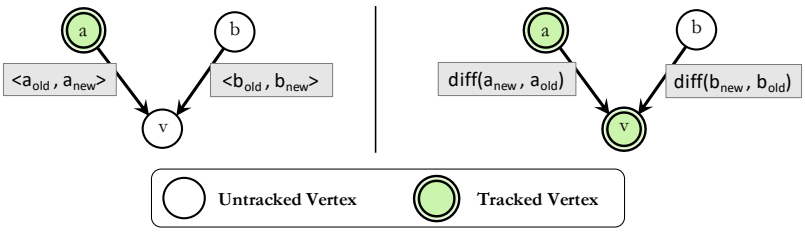
4.3.2 Propagating Differences upon Graph Mutation

When the graph structure mutates, the incremental computation must correctly propagate changes throughout the available intermediate states to compute final results. To retain BSP guarantees at the end of each iteration, our selective incremental processing propagates values iteration-by-iteration.

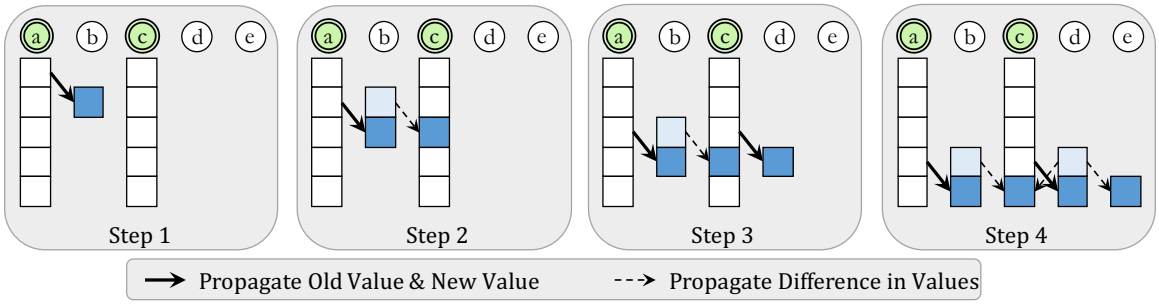
With only a subset of intermediate states available, processing for different vertices is handled differently. For tracked vertices, the intermediate states are incrementally refined in-place as processing progresses through iterations. Whereas for untracked vertices, a single

Algorithm 2 Selective Incremental Processing

```
1: parallel for e ∈ E s.t. target(e) is untracked do
2:   Activate source(e) for propagation to untracked targets
3: end parallel for

4: for i ∈ [1...k] do
5:   /* Propagate changes directly resulting from mutations */
6:   parallel for e ∈ mutated edges s.t. target(e) is tracked do
7:     Propagate old change if e is added; otherwise retract old change
8:     Activate target(e) for vertex computation
9:   end parallel for
10:  parallel for e ∈ E mutated edges s.t. target(e) is untracked do
11:    Propagate old change if e is removed; otherwise retract old change
12:    Activate target(e) for vertex computation
13:  end parallel for

14:  /* Propagate transitive changes from active vertices */
15:  parallel for e ∈ E s.t. target(e) is tracked and (source(e) is active or e is mutated
     edge) do
16:    Propagate difference between old change and new change
17:    Activate target(e) for vertex computation
18:  end parallel for
19:  parallel for e ∈ E s.t. target(e) is untracked and source(e) is active do
20:    Propagate old change and new change
21:    Activate target(e) for vertex computation
22:  end parallel for

23:  /* Compute vertex values and differences to push in next iter */
24:  parallel for v ∈ active tracked vertices do
25:    Merge difference in v's intermediate state
26:    Compute v's old value and new value
27:    Activate v for propagation if difference in value changes is not ∅
28:  end parallel for
29:  parallel for v ∈ active untracked vertices do
30:    Merge old change in v's old value and new change in v's new value
31:    Compute v's old value and new value
32:    Activate v for propagation if difference in value changes is not ∅
33:  end parallel for
34: end for
```



(a) Values propagated based on whether the target vertex is tracked (highlighted) or untracked. Untracked vertices receive the old and new values, whereas tracked vertices directly receive difference in values.



(b) Selective incremental processing with a and c as tracked vertices, for addition of a new edge from a to b. Intermediate values for untracked vertices (b, d and e) get computed, propagated, and thrown out as execution progresses.

Figure 4.3: Value Propagation in Selective Stateful Model

vector is maintained to hold their latest values as processing progresses through iterations (similar to computing from scratch without intermediate states). The iterative execution is summarized in Figure 4.3. In each iteration, the values propagated across edges are based on whether the target vertices are tracked or untracked. If the target vertex is tracked, then the difference in value is propagated along its incoming edge similar to the traditional dependency-driven incremental processing [26]. On the other hand, if the target vertex is untracked, then both the old value (from before graph mutation) and the new value (resulting from graph mutation) are propagated along the edge. This allows the target vertex to compute the necessary differences for its outgoing neighbors in the subsequent iteration.

Algorithm 2 shows how our selective incremental processing propagates values upon graph mutation. In each iteration, the differences directly resulting from graph mutation are propagated (lines 6-13), and then the resulting differences are propagated for the tracked and untracked vertices (line 15-22). The tracked vertices acquire the difference between the previous value change and the new value change. The untracked vertices, on the other hand, receive two values: the previous change and the new change. This allows the untracked vertices to recompute the old aggregation along with the updated aggregation. Once the values arrive at the required active vertices, their vertex values are computed to identify the differences to be propagated in the next iteration (lines 24-33).

Algorithm 3 PageRank using Selective Stateful Iterative Model

```
1: function addChange(AggrType * nghSum, DeltaType delta)
2:   AtomicAdd(nghSum, delta)
3: end function
4: function removeChange(AggregationType * nghSum, DeltaType delta)
5:   AtomicSub(nghSum, delta)
6: end function
7: function computeVertexValue(AggrType nghSum)
8:   return nghSum × 0.85 + 0.15
9: end function
10: function computeDelta(DeltaType newDelta, DeltaType oldDelta)
11:   return newDelta − oldDelta
12: end function
13: function computeOutDelta(VId v, VValType nextV, VValType currV, Graph g)
14:   return (nextV − currV)/g.degree[v]
15: end function
16: function checkConvergence(VId v, VValType nextV, VValType currV)
17:   return | nextV − currV |> threshold
18: end function
```

As we can see on lines 15 and 19, operations on edges based on their target being tracked or untracked are directly invoked in parallel without any checks per edge, mainly because of the optimized graph layout described above that separates the edges. Moreover, since the selective incremental processing recomputes the vertex values to identify differences, our model tracks only the aggregation values as intermediate states (i.e., it does not track the intermediate vertex values, which is also maintained as part of intermediate state in the traditional dependency-driven incremental refinement [26]).

Implementation

Implementing algorithms with selective stateful iterative model is done by expressing the sub-computations in change-driven or differential manner. We expose a simple API to propagate changes and incrementally recompute the states. Algorithm 3 shows the PageRank computation using our API.

In a given iteration, *addChange*() and *removeChange*() incrementally aggregate the (direct or transitive) differences in values; for PageRank this happens via addition and subtraction operations. These functions are invoked while propagating the changes resulting from mutations (lines 5-13 in Algorithm 2) as well as transitive changes in subsequent iterations (lines 15-22 in Algorithm 2). The *computeVertexValue*() function computes vertex values using the aggregation values that are incrementally adjusted (invoked on lines 26 and 31 in Algorithm 2). The *computeDelta*() function is used to propagate the difference between the old delta and new delta to the tracked vertices which is generated by *computeOutDelta*() (invoked on line 16 in Algorithm 2). At the end of each iteration,

checkConvergence() is used to identify whether changes have converged to avoid further propagations.

Since our API captures operations on vertices and edges (which are internally invoked in vertex-parallel and edge-parallel manner), they enable expressing a wide range of graph applications. To capture the value changes at a finer level, the separation of *computeDelta*() and *computeOutDelta*() are the only modifications compared to APIs exposed by existing frameworks.

Chapter 5

Minimal Stateful Iterative Model

In this chapter, we develop the *minimal stateful iterative* model that aggressively eliminates the tracking of intermediate state by specializing the incremental processing for certain graph algorithms. Specifically, our model will directly operate on 'value differences' without reconstructing the intermediate states so that effects of mutations get propagated only as value differences throughout the iterations. We first summarize the properties of algorithms that enable this specialization, and then discuss the details of the minimal stateful iterative model.

5.1 Distributive Update Property

Computations in graph algorithms can be modelled as:

$$val(v) = A\Big(\bigoplus_{(u,v)\ \in\ E} \Big(S(val(u))\Big)\Big)$$

where $\oplus$ is the the aggregation function that combines incoming values to a vertex, S is the function that transforms the source's value to be aggregated (analogous to scatter operation in [14]), and A is the vertex function that computes the vertex value using the aggregation result. For instance, in PageRank $\oplus$ is the *sum* operation, S is the function that divides the rank value with outdegree (i.e., `pr(u) / out_degree(u)`), and A is the linear equation that computes rank value using the result of $\oplus$ and damping factor (i.e., `(1-d) + d * sum`). The distributive update property states that the computation of vertex value can be distributed as sub-computations over its incoming neighbors' values, i.e.,

$$A\Big(\bigoplus_{k\in\{w,x,y,z\}} \Big(S(k)\Big)\Big) = \underset{k\in\{\{w,x\},\{y,z\}\}}{\gamma} \Big(\alpha\Big(\bigoplus_{k'\in k} (S(k'))\Big)\Big)$$

where γ and α are functions derived from A. This property is important because it allows directly computing the difference in the target value from the difference in the source value without reconstructing $\oplus(S(*))$. This allows our minimal stateful iterative model to

Algorithm 4 Incremental Processing with Minimal State

```
1: for i ∈ [1...k] do
2:   /* Propagate old values */
3:   parallel for e ∈ mutated edges do
4:     Propagate old value if e is added; otherwise retract old value
5:     Activate target(e) for vertex computation
6:   end parallel for

7:   /* Propagate transitive changes from active vertices */
8:   parallel for e ∈ E s.t. source(e) is active or e is mutated edge do
9:     Propagate change
10:  end parallel for
11:  parallel for v ∈ active vertices do
12:    if v is tracked then
13:      Merge change in v's intermediate state
14:    end if
15:    Activate v for propagation if change is not ∅
16:  end parallel for
17: end for

18: /* Compute final vertex values */
19: parallel for v ∈ V do
20:   Merge change in vertex value with old vertex value
21: end parallel for
```

aggressively reduce the intermediate state by simply not tracking the results from $\oplus$. For instance, in our PageRank example if a source's rank value changes from $\texttt{u}_1$ to $\texttt{u}_2$, then the change in value of the destination vertex v gets directly computed as $\texttt{d * (u}_2\texttt{ - u}_1\texttt{) / out\_degree(u)}$. Note this does not require explicitly reconstructing the value of `sum` variable for v.

It is important to note that the distributive update property described above is different from just the aggregation operation being distributive. While most of the aggregation operations are distributive (which enables edge parallel operations, as well-known in prior research), the distributive update property also requires the vertex functions to be distributive. For instance, the PageRank computation satisfies this property since its linear equation only operates on the aggregation value and constants, and hence, any change in rank value of source vertex can be directly incorporated in the destination value. On the other hand, even though algorithms like Collaborative Filtering [55] and Multi-Modality Learning [43] have *sum* aggregation (which is distributive), their vertex functions are not distributive since they involve operations like *normalization* and *vector product*. Table 5.1 shows various graph algorithms whose computation satisfies the distributive update property, along with those whose computations violate the property. While many algorithms satisfy the property, operations like matrix inverse, multiplication/division and value normalization limit the overall computation from being distributive.

Algorithm	Satisfies Distributive Update Property?	Reason for Violation
PageRank	✓	-
Co-Training Expectation Maximization	✓	-
Katz Centrality	✓	-
Async. Algorithms like: Breath First Search, Shortest Paths, Connected Components, Widest Paths, Minimal Spanning Tree	✓	-
Collaborative Filtering	✗	Matrix inverse & multiplication
Circuit Simulation	✗	Division
Label Propagation	✗	Value Normalization
Multi-Manifold Ranking	✗	Value Normalization
Multi-Modality Learning	✗	Value Normalization

Table 5.1: Algorithms whose computations satisfy (✓) or violate (✗) the distributive update property.

5.2 Tracking Minimal Vertex State

While the differences can be propagated without computing the intermediate states at each iteration, these differences need to be grounded w.r.t. some basis so that they are meaningful. Hence, we track the earliest intermediate state that initiates the incremental computation when graph structure mutates. These earliest intermediate states correspond to the states of the mutation points (e.g., vertices whose edges got mutated) since those points start propagating the changes directly based on the specific edge/vertex that gets added/deleted. Apart from the earliest states, no other intermediate state is captured since the computations purely operate on differences to propagate through the rest of the iterations.

Tracking the earliest intermediate states requires knowing the mutation points upfront, which may not always be possible. However, application-specific insights like mutations occurring at certain important vertices, or what-if queries based on certain regions of the graph can help determine the subset of intermediate state that must to be tracked.

5.2.1 Incremental Processing

When graph structure mutates, incremental computation is performed in iteration-by-iteration manner by purely operating on differences. Algorithm 4 shows how the differences are identified and propagated. Unlike the selective incremental processing technique, the distributive update property enables straightforward propagation of differences. The mutated edges propagate and retract the old values (lines 3-6), and their target vertices

Algorithm 5 PageRank using Minimal Stateful Iterative Model

```
1: function addChange(AggrType* nghSum, DeltaType delta)
2:   AtomicAdd(nghSum, delta)
3: end function
4: function removeChange(AggregationType* nghSum, DeltaType delta)
5:   AtomicSub(nghSum, delta)
6: end function
7: function computeVertexValue(AggrType nghSum)
8:   return nghSum × 0.85 + 0.15
9: end function
10: function computeDelta(DeltaType newDelta, DeltaType oldDelta)
11:   return newDelta − oldDelta
12: end function
13: function computeOutDelta(VId v, VValType nextV, VValType currV, Graph g)
14:   return (nextV − currV)/g.degree[v]
15: end function
16: function accVertexVals(VValType oldVal, VValType changeVal)
17:   return oldVal + changeVal
18: end function
```

compute the differences. These differences are further propagated in subsequent iterations (line 9). If the target vertex is tracked, the differences are merged in the intermediate state as computation progresses. In the end, the cumulative differences are incorporated with the vertex values to generate the final result (line 20).

Implementation

The algorithms for minimal stateful iterative models are expressed using the API similar to that for the selective stateful iterative model. Algorithm 5 shows the PageRank example. The *addChange*() and *removeChange*() functions incrementally incorporate differences to aggregation value and are used to apply direct and transitive changes to the vertices (used on lines 4 and 9 in Algorithm 4). Unlike the selective stateful iterative model, only changes caused by graph mutation should be propagated, and the old vertex values do not have to be computed. The *computeVertexValue*() function computes the vertex value based on the aggregated value. The *computeDelta*() and *computeOutDelta*() functions allow to compute the differences in vertex values for propagating to the neighbors. The *accVertexVals*() function aggregates the differences in vertex values across all the iterations till the end, which is used to obtain the final result after all the iterations have finished. This function is used on line 20 in Algorithm 4 to merge the cumulative changes and produce final vertex values.

Chapter 6

Evaluation

In this chapter, we thoroughly evaluate our memory-efficient stateful iterative models and compare their performance with the dependency-driven incremental processing model from GraphBolt [26] (which delivers high performance at the cost of high memory consumption). Specifically, we answer the following questions:

1. How effective is our selective stateful iterative model in controlling the memory footprint?
2. How does the performance of our selective stateful iterative model vary as the number of vertices being tracked changes?
3. How effective is our minimal stateful iterative model in maintaining a small memory footprint while still delivering high performance?
4. How do our memory-efficient stateful iterative models perform when processing a large number of simultaneous graph mutations?

6.1 Implementation Details

We implemented our memory-efficient stateful iterative models in the GraphBolt system for two main reasons: first, it allows our models to utilize the efficient implementation of the underlying framework (e.g., parallelization strategy, atomics, frontiers, etc.); and second, it enables direct performance comparison of our models with GraphBolt's existing execution model.

We implemented the optimized graph layout for the adjacency list representation (discussed in Section 4.3) to replace the existing adjacency list data structure. The intermediate states for selected subset of vertices are tracked in their respective arrays. The state arrays get allocated vertically (per vertex) instead of horizontally (per iteration) so that arrays for untracked vertices are not allocated (hence reducing memory footprint), while at the same time the intermediate states for tracked vertices get addressed without using any hashmap.

Graph	Vertices	Edges	Graph Size	
			Without Final State	With Final State
SK-2005 (SK)	50.6M	2B	13.8GB	18-28GB
TwitterMPI (TT)	52.6M	2B	14.2GB	19-30GB
Friendster (FT)	68.3M	2.5B	18.74GB	24-38GB
UK-2007-05 (UK)	105M	3.7B	27.75GB	36-59GB
UK-union (UN)	133M	5.5B	36.2GB	48-80GB
Clueweb (CWB)	978.4M	42.5B	130GB	197-240GB

Table 6.1: Real-world graphs used in experiments[4, 5, 3]

6.2 Experimental Setup

We use eight synchronous graph algorithms. PageRank (PR) [33] computes the importance of web-pages based on incoming links to those pages. Collaborative Filtering (CF) [55] is a context-based technique used in recommender systems to classify associated items while Co-Training Expectation Maximization (CoEM) [32] is a semi-supervised learning algorithm for named object identification. Katz Centrality (KC) [19] measures the centrality as the relative degree of influence in the graph. Multi-Manifold Ranking (MMR) [52] is a ranking method that uses multiple image manifolds each constructed using a different image features. Multi-Modality Learning (MML) [43] and Label Propagation (LP) [56] are learning algorithms that disperse labels from a subset of vertices to assign label to the rest of the graph. Circuit Simulation (CS) [20] simulates flow in a circuit by solving partial differential equation.

The LP, MMR and MML algorithms compute vector of features for each vertex, whereas the remaining algorithms operate on scalar vertex values except for CF which operates on two factors per vertex. Computations in PageRank, CoEM and KC follow the distributive update property (described in Section 5.1), and hence we evaluate our minimal stateful iterative model with these three benchmarks. As mentioned in Table 5.1, computations in the remaining benchmark do not follow the distributive update property due to the complex sub-operations they involve: specifically, LP, MMR and MML normalize the feature vectors in every iteration, CF computes matrix inverse, and CS uses division on its aggregation values.

Table 6.1 lists the six real-world input graphs used for evaluation. Similar to [26, 39], we obtained an initial fixed point when 50% of edges were loaded, and streamed in the remaining edges to model edge insertions, while randomly sampled edges from the loaded graph were used for edge deletions. To eliminate the effects of locality, we shuffled the edges while forming our edge streams. For CWB graph, initial fixed point was obtained with 7% of edges so that at least stateless execution could successfully execute. The algorithms that operate on vectors consume high memory, and as expected, the memory footprint increases as the vector size increases. Unless otherwise stated, we use 10 features for LP, MMR and MML so that the GraphBolt baseline could hold the intermediate state without running out of memory, and 2 features are used for CWB graph to ensure that stateless execution could

successfully execute. Similar to [26], we run all algorithms for 10 iterations. Unless otherwise stated, we apply 10K edge mutations to evaluate how quickly our models compute the final result; we also vary the mutation batch size from a single mutation to up 10 million edge mutations. Our implementations produce correct final results, and we verified by comparing them with the results produced by the stateless BSP executions that start from scratch. All experiments were performed on Oracle Cloud VM.Standard2.24 shape containing Intel(R) Xeon(R) 8167M processor with 24 physical cores (48 threads) and 320GB main memory running 64-bit Ubuntu 18.04.

Throughout the evaluation, we use the following notations for different executions:

- **Selective-k%:** this is our selective stateful iterative model that tracks k% of total vertices.
- **Minimal:** this is our minimal stateful iterative model.
- **GraphBolt:** this is GraphBolt's execution as baseline, which tracks the intermediate state for all vertices.
- **Stateless:** this baseline does not track any intermediate state, and recomputes values from scratch upon graph mutation.

6.3 Performance of Selective Stateful Model

Figures 6.1, 6.2 and 6.3 show the memory footprint and execution time for our selective stateful iterative model when tracking the intermediate state for 20%, 40%, 60% and 80% of the vertices. The figure also compares the performance with GraphBolt and stateless executions. Figure 6.4 summarizes similar results for CWB graph. As we can see, our selective stateful iterative model is effective in controlling the memory footprint by tracking only the selected subset of vertices. For instance, it consumes 35-70% less memory than GraphBolt when tracking only 20% of vertices, while at the same time delivering 15-83% of the performance gains provided by GraphBolt over the stateless execution. In fact, GraphBolt runs out of memory for certain cases while the selective executions end up successfully executing and delivering high performance.

By tracking more intermediate states the memory footprint increases and the execution time decreases mainly because the intermediate state helps in incremental refinement; this is visible as decreasing number of edge operations in Figure 6.5 for FT graph as the number of tracked vertices increase from 20% to 80%. Since stateless execution does not track any intermediate state, its memory footprint is the lowest while the execution time is highest; on the other extreme, since GraphBolt tracks intermediate state for all vertices, its memory footprint is the highest while its execution time is the lowest. Our selective stateful iterative

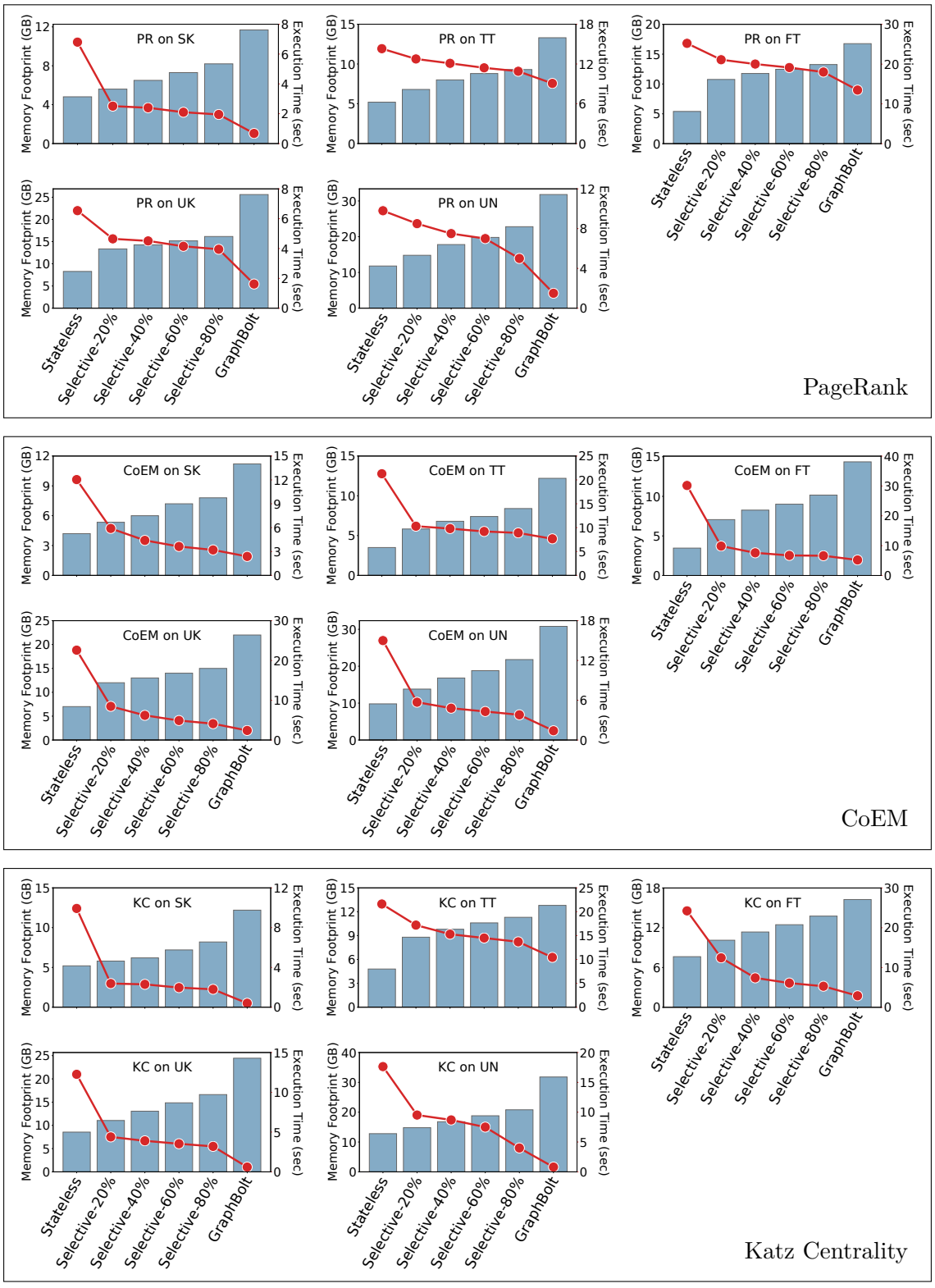


Figure 6.1: Performance of our selective stateful iterative model compared to the stateless iterative model and the stateful iterative model from GraphBolt for PR, CoEM and KC. The memory footprints (in GB) are shown as bars (left y-axis) and the execution times (in seconds) are shown as points (right y-axis).

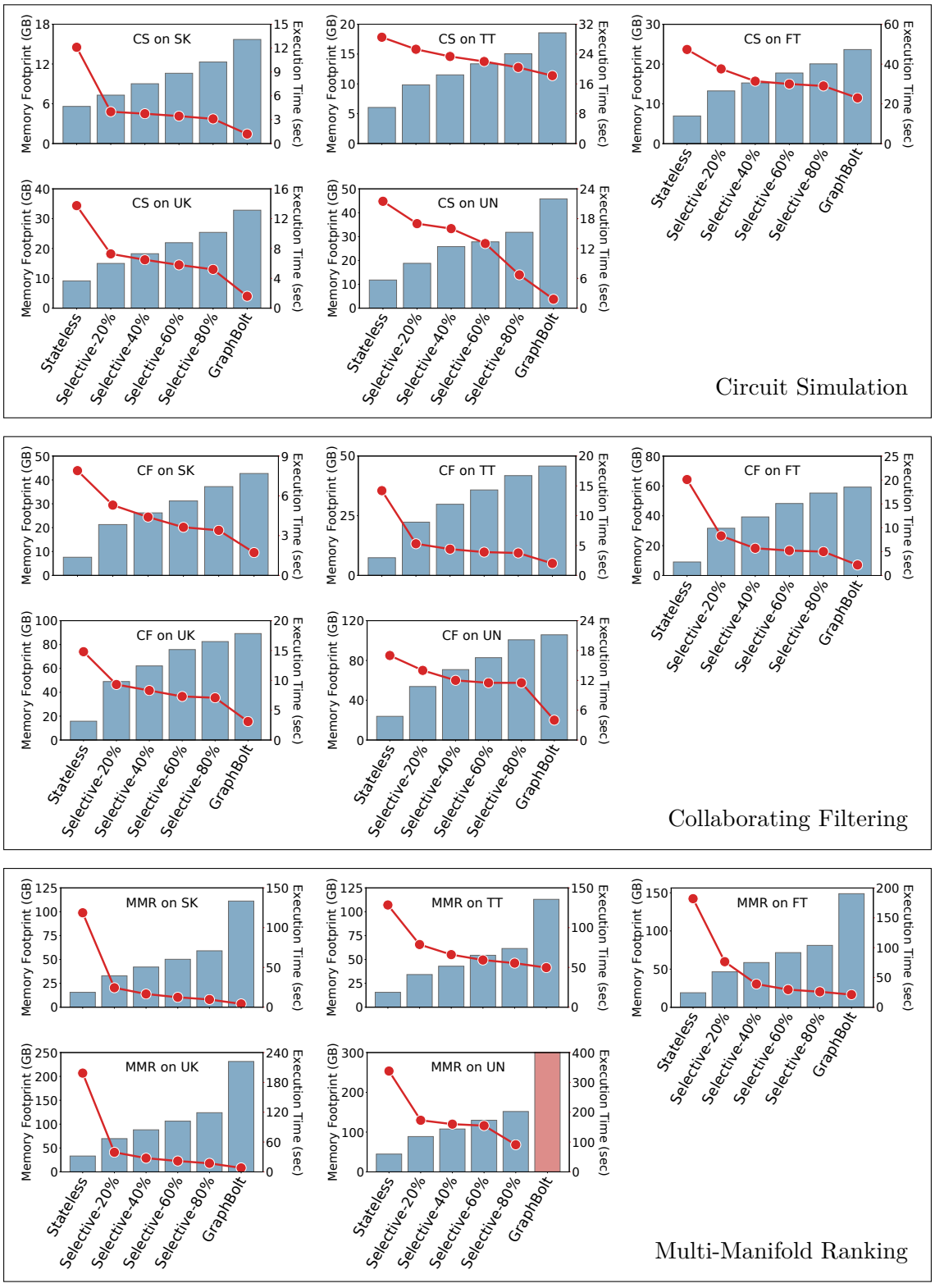


Figure 6.2: Performance of our selective stateful iterative model compared to the stateless iterative model and the stateful iterative model from GraphBolt for CS, CF and MMR. The memory footprints (in GB) are shown as bars (left y-axis) and the execution times (in seconds) are shown as points (right y-axis). Red bar indicates the execution ran out of memory.

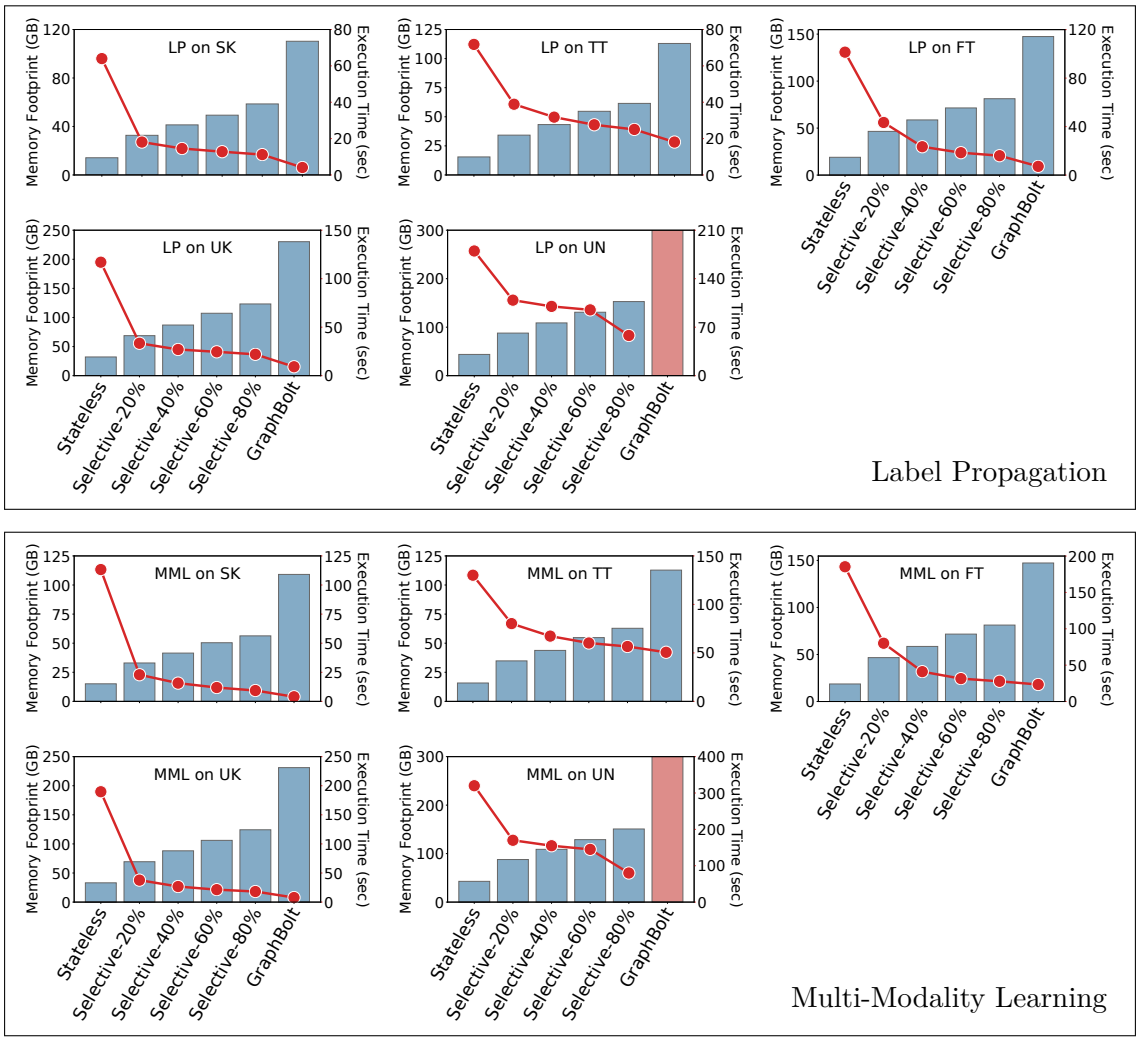


Figure 6.3: Performance of our selective stateful iterative model compared to the stateless iterative model and the stateful iterative model from GraphBolt for LP and MML. The memory footprints (in GB) are shown as bars (left y-axis) and the execution times (in seconds) are shown as points (right y-axis). Red bar indicates the execution ran out of memory.

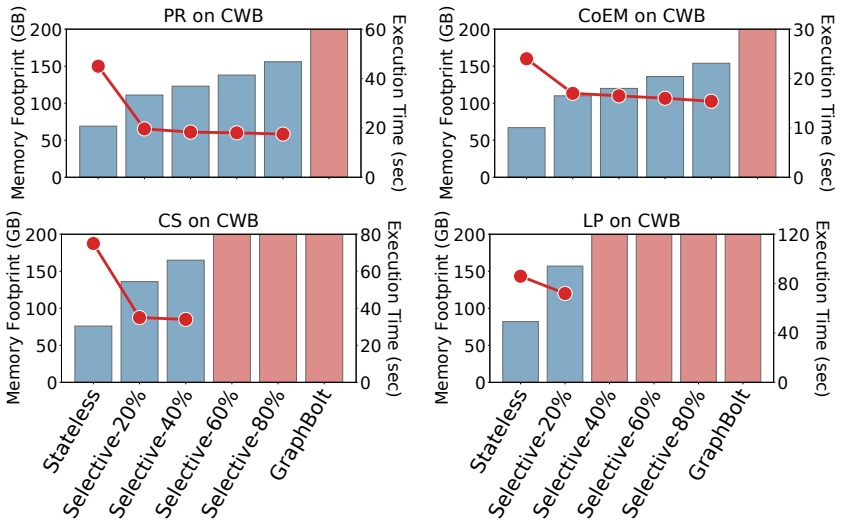


Figure 6.4: Performance of our selective stateful iterative model on CWB graph compared to the stateless iterative model and the stateful iterative model from GraphBolt. The memory footprints (in GB) are shown as bars (left y-axis) and the execution times (in seconds) are shown as points (right y-axis). Red bar indicates the execution ran out of memory.

model trades off memory footprint for more computation, and hence delivers performance between the two extremes.

We observe that the increase in memory footprint from stateless to selective-20% is higher compared to the increase between consecutive selective variants (e.g., between selective-20% and selective-40%). This is because the selective incremental processing computes both the new value (after mutation) and the old value (prior to mutation) so that differences can be propagated from untracked values; holding these values in memory adds to the memory footprint, which is an overhead that stateless executions do not incur.

For a given graph, the memory footprint depends on the size of intermediate state in the benchmark. Hence, for each graph the memory footprints are lower for PageRank and CoEM since they operate on scalar values, while the footprints are higher for LP, MMR and MML due to their use of feature vectors. This is also the reason why GraphBolt runs out of memory for only LP, MMR and MML on UN graph; whereas on CWB graph only certain executions of selective variants run successfully.

The execution time, on the other hand, is dependent on how the values propagate across iterations which is dependent on the graph algorithm and the structure of input graph. Therefore, the performance benefit with saving selective intermediate state is different for different cases. For instance, selective-20% is 5× faster compared to stateless for MMR on UK, whereas it is only 1.63× faster for MMR on TT. On the other hand, selective-20% is 2.4× faster compared to stateless for CF on FT, but only 1.2× faster for CF on UN.

For most of the cases, we observe that tracking intermediate state for only 20% of vertices achieves 15-83% of performance gains that are achieved by GraphBolt. For instance

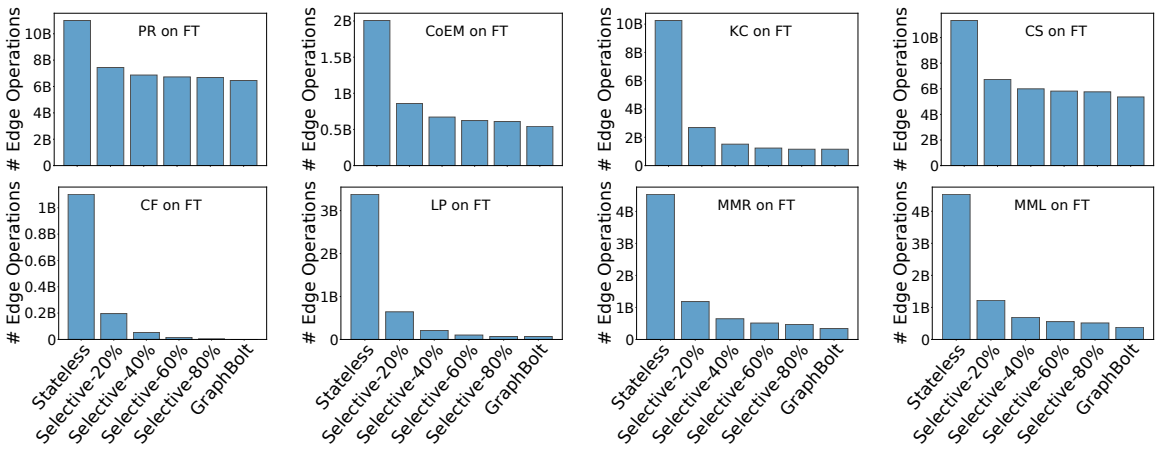


Figure 6.5: Number of edges operations executed by selective stateless iterative model, compared with stateless iterative model and stateful iterative model from GraphBolt.

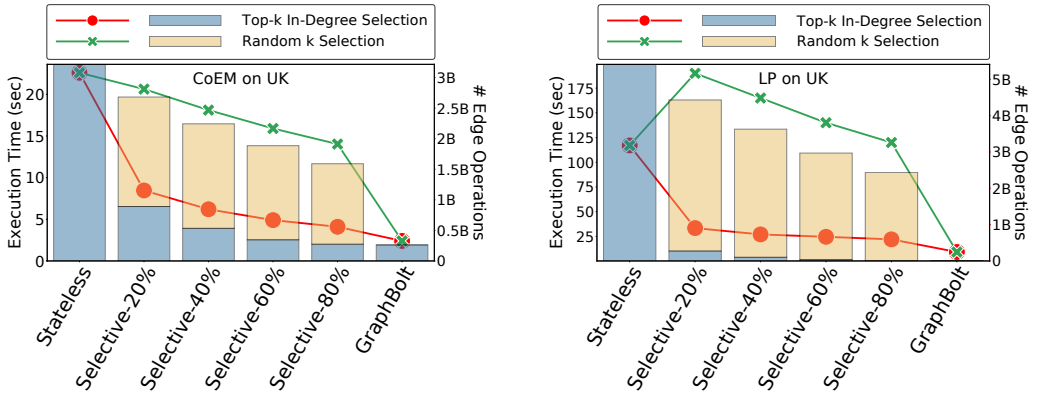
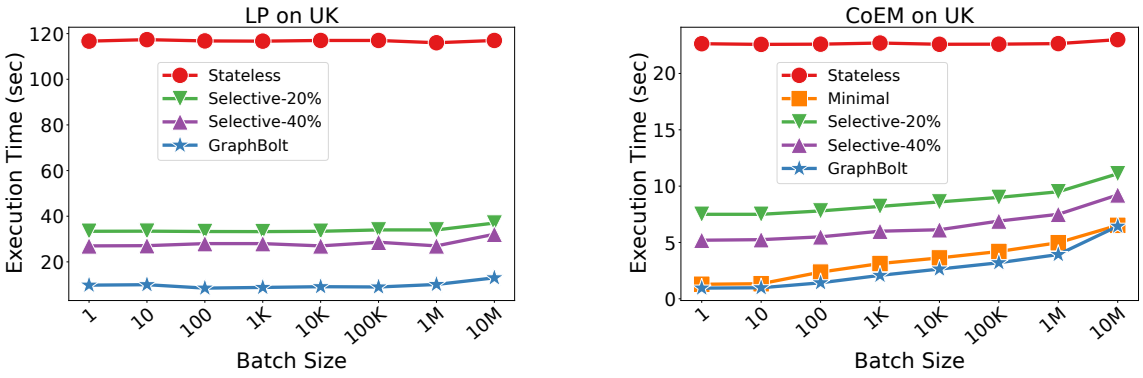


Figure 6.6: Performance of selective stateful iterative model with randomly selected vertices to be tracked compared with our in-degree based top-k selection heuristic. The execution times (in seconds) are shown as points (left y-axis) and the number of edge operations are shown as bars (right y-axis).

selective-20% on MMR achieves 1.6-5× compared to stateless executions across different graphs, whereas GraphBolt achieves 2.6-24.4× compared to stateless execution. This is mainly because our model tracks the high-degree vertices that demand more computation if their states are not available, and hence incremental processing for those high-degree vertices ends up achieving high performance benefits. This is a benefit especially for skewed graphs since the memory consumption for selective-20% is much less compared to GraphBolt, while at the same time the performance gains are high. As expected, the performance gains from incremental processing reduce as the number of tracked vertices increase.

We measured the effectiveness of our in-degree based top-k selection strategy by comparing the performance with a stochastic selection strategy. Figure 6.6 shows the execution time and number of edges processed when vertices to be tracked are selected at random compared to when using our highest in-degree heuristic. As we can see, tracking randomly selected vertices ends up processing more number of edges (and hence delivers lower perfor-



(a) Label Propagation (b) Co-Training Expectation Maximization

Figure 6.7: Execution times (in seconds) for different iterative models across varying number of mutations per batch.

mance) compared to that processed when high-degree vertices are tracked. This is because tracked vertices save computation for their incoming edges, and hence high in-degree vertices reduce more computation compared to low in-degree vertices. With random set of vertices being tracked, the in-degrees of those tracked vertices are often not very high. In fact, we observe that the stateless execution for LP outperforms the selective stateful model with random selection as the little gains provided by tracking random vertices do not fully offset the overheads of the selective stateful model (i.e., propagating multiple values, and separately propagating to tracked and untracked vertices).

Finally, for cases like CF we observe that the difference in execution times between selective-80% and GraphBolt is higher compared to the difference between consecutive selective variants (e.g., between selective-60% and selective-80%). This is again because the selective incremental processing computes both the old values and the new values whereas GraphBolt directly computes the value changes. The effects of these additional computations become more visible for CF since it has relatively expensive vertex computations (involving a matrix inverse operation).

6.3.1 Scaling with Mutation Batch Sizes

Figure 6.7a shows the performance of our selective stateful iterative model with 20% and 40% tracked vertices as the mutation batch size increases from a single mutation to up 10 million edge mutations. The memory footprints of the stateful iterative models remain same as in Figures 6.1, 6.2 and 6.3 since they are mainly dependent on the number of tracked vertices. However, we observe that the amount of computation performed increases as more mutations get simultaneously applied. Hence, the execution time for both, selective stateful model as well as GraphBolt increases as mutation batch size increases. Since the stateless execution simply recomputes from scratch, its execution time increases very slowly across different mutation batch sizes.

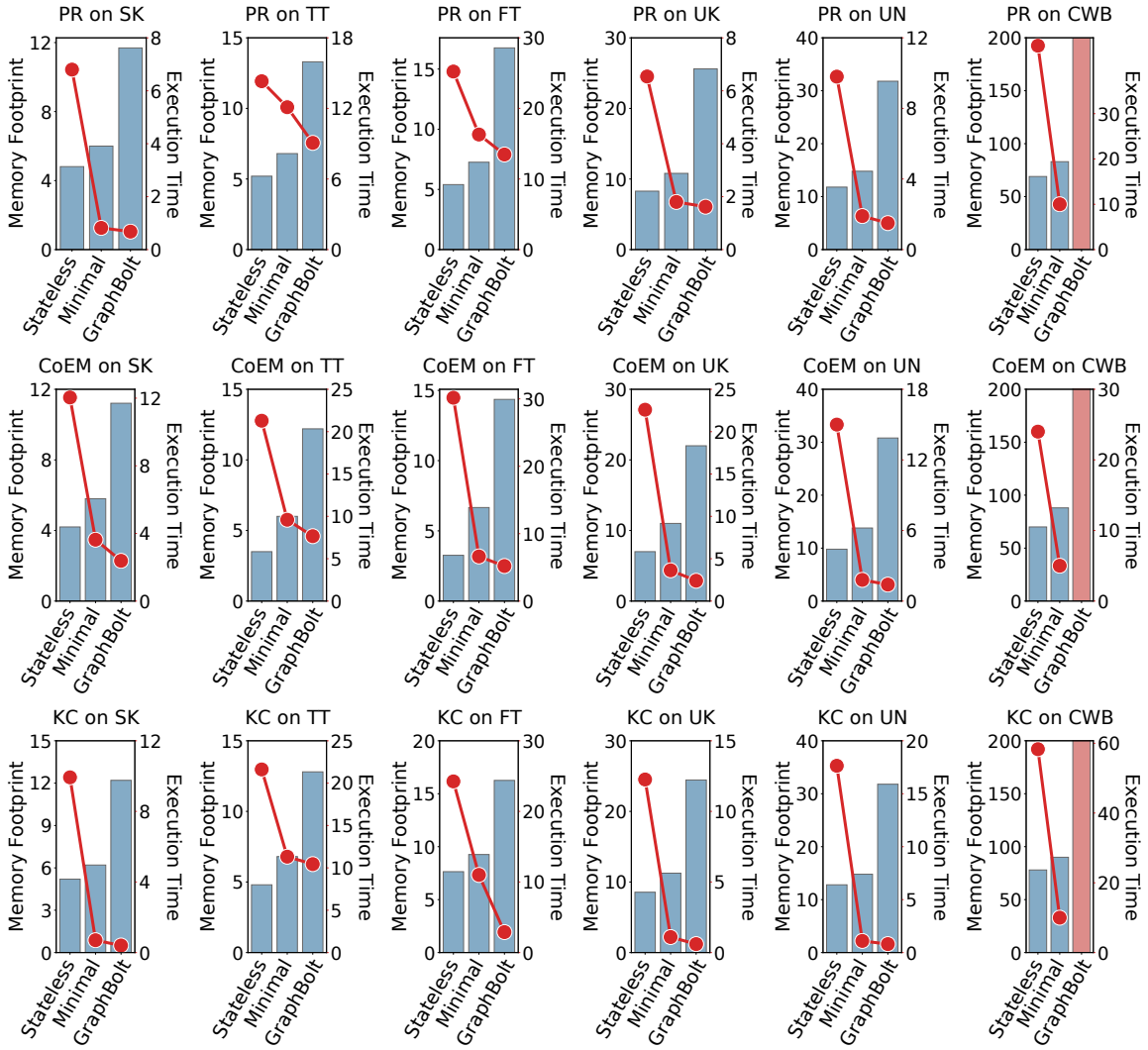


Figure 6.8: Performance of our minimal stateful iterative model compared to the stateless iterative model and the stateful iterative model form GraphBolt. The memory footprints (in GB) are shown as bars (left y-axis) and the execution times (in seconds) are shown as points (right y-axis). Red bar indicates the execution ran out of memory.

6.4 Performance of Minimal Stateful Model

Figure 6.8 shows the memory footprint and execution times for our minimal stateful iterative model along with GraphBolt and stateless executions on PageRank, CoEM and Katz Centrality. Since the minimal stateful iterative model only tracks the earliest intermediate states that form the basis for differences, its memory footprint is 1.4-2.4× smaller compared to GraphBolt and only 1.1-1.3× higher than stateless execution. Moreover, the incremental computation in the minimal stateful iterative model purely operates on value differences, and hence, delivers high performance. Our minimal stateful iterative model is 1.1-8.2× faster than stateless executions, which results in 65-90% of the benefits delivered by GraphBolt.

Comparing with performance results from Figure 6.1 and Figure 6.4, we observe that minimal stateful iterative model outperforms selective-80% in most of the cases while consuming lesser memory than that required by selective-20%.

6.4.1 Scaling with Mutation Batch Sizes

Figure 6.7b shows the performance of the minimal stateful iterative model for CoEM as the mutation batch size increases from a single mutation to 10 million edge mutations. Similar to selective model, the execution time for the minimal stateful iterative model increases as the number of simultaneous mutations increases; nevertheless, it is 3.5-17.4× faster than the stateless execution.

Unlike the selective model, the memory footprint of our minimal stateful iterative model is sensitive to the number of updates. As the mutation batch size increased from 1 to 10M, the memory footprint of our minimal stateful iterative model increased from 0.5GB (1.06× higher than stateless, and 2.9× lower than GraphBolt) to 2GB (1.3× higher than stateless, and 2.4× lower than GraphBolt).

Chapter 7

Conclusions & Future Directions

We presented two memory-efficient stateful iterative models for incremental processing of streaming graphs.

The Selective Stateful Iterative Model tunes the memory footprint based on the available main memory by selecting only a limited portion of the intermediate state to be tracked throughout execution. Upon graph mutation, our selective stateful model incrementally updates the maintained intermediate state for tracked vertices, and computes the values of untracked vertices from scratch iteration-by-iteration.

The Minimal Stateful Iterative Model further reduces the memory footprint by exploiting the distributive update property in certain graph algorithms to eliminate tracking of intermediate states. Our model directly computes 'value differences' without constructing the intermediate states, and propagates those differences throughout the iterations. In the end, the accumulated differences are integrated with the final vertex value to produce the final values.

Our models are general and can be incorporated into any streaming graph processing system. We incorporated both the models in GraphBolt [26] and showed that they significantly reduce the memory footprint while still retaining most of the performance benefits of the traditional stateful iterative model in GraphBolt. This also allowed our models to scale on larger graphs that could not be handled by the traditional models.

7.1 Future Directions

Although our memory-efficient models are effective in limiting the memory footprint, they can be extended to aggressively reduce the memory consumption in several ways.

Approximate Computing

Both of our memory-efficient models deliver strong correctness guarantees so that the final results are consistent with those generated by a bulk synchronous parallel execution from scratch. By relaxing the accuracy requirement, tracking and updating of certain intermediate

can be skipped and, upon graph mutation, the processing can be continued from final vertex values. Such a technique would eliminate the need to reconstruct the old states for untracked vertices which would directly reduce the amount of the edge operations and hence reduce the overall execution time.

Expressing Distributive Computations

The distributive update property is useful to aggressively reduce the memory consumption using our minimal stateful iterative model. While several algorithms have operations that satisfy the distributive update property, they also have other operations that do not directly satisfy the property. For instance, *normalization* which is a common sub-operation across in our graph benchmarks is not distributive. A potential future direction can be to rewrite such operations so that they follow the distributive property, and allow the entire algorithm to leverage the minimal stateful iterative model. This can be done by analyzing the sub-operations to understand their behaviors, and developing analogous operations that retain the same behaviors.

Compression

Compression techniques [5, 10, 23, 41] can also be utilized along with our memory-efficient models. Since compression techniques generally deal with data layout, they remain unaware of the processing semantics, and hence can be applied in tandem with our proposed models.

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